Solutions to JEE Advanced Booster Test - 3 | 2024 | Code A

[PHYSICS]

1.(A) Equation of trajectory:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

The point $(6,-4)$ must satisfy this equation. Therefore:

$$-4 = 6\tan\theta - \frac{(10)(6)^2}{2(6)^2}\left(1+\tan^2\theta\right)$$

$\Rightarrow \quad 5\tan^2\theta - 6\tan\theta + 1 = 0 \quad \Rightarrow \quad \tan\theta = 1, \frac{1}{5}$

2.(A) Considering forces in horizontal direction on wedge

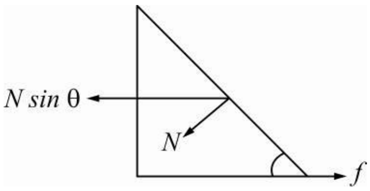
As wedge is not moving in horizontal direction

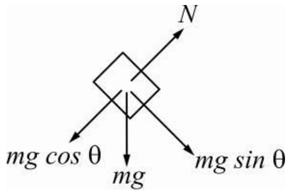
$f = N\sin\theta$

Here $N = mg\cos\theta$

$f \equiv mg\cos\theta\sin\theta$

$\Rightarrow \quad 2\times 10\frac{1}{2}\times\frac{\sqrt{3}}{2} = 5\sqrt{3}N$





3.(A) Radius of curvature at highest point

$\Rightarrow \quad mg = \frac{m(u\cos\theta)^2}{r} \quad \Rightarrow \quad r = \frac{u^2\cos^2\theta}{g}$

Radius of curvature > Maximum height,

$$\frac{u^2\cos^2\theta}{g} > \frac{u^2\sin^2\theta}{2g} \quad \Rightarrow \quad \theta < \tan^{-1}\sqrt{2}$$

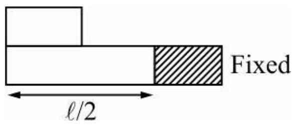
Radius of curvature < Maximum height

$$\frac{u^2\cos^2\theta}{g} < \frac{u^2\sin^2\theta}{2g} \quad \Rightarrow \quad \theta > \tan^{-1}\sqrt{2}$$

4.(A) Acceleration of combined block $= \frac{F}{2M}$

Velocity gained by upper block before collision $(V) = \sqrt{2\times\frac{F}{2M}\times\ell} = \sqrt{\frac{F\ell}{M}}$

$V = \sqrt{\frac{F\ell}{M}} \qquad V_f = 0$



Retardation, $a = \mu g = \frac{V^2}{2(\ell/2)}$

$\mu = \frac{F}{Mg}$

5.(C) Given, $a_t = -\alpha s^2$

or, $\frac{dv}{ds} \times \frac{ds}{dt} = -\alpha s^2$; $v\frac{dv}{vs} = -\alpha s^2$

Suppose particle will come to rest after '*n*' revolution

So distance travelled $2\pi Rn$

$\Rightarrow \int_{V_0}^{0} v\, dv = -\int_{0}^{2\pi Rn} \alpha\, s^2\, ds$; $\frac{-V_0^2}{2} = \frac{-\alpha s^3}{3}\Big|_0^{2\pi Rn}$ $\Rightarrow$ $n = \frac{1}{2\pi R}\left(\frac{3V_0^2}{2\alpha}\right)^{1/3}$

6.(AC) Acceleration: $a(t) = Pt\,\hat{i} + Q\,\hat{j}$

Velocity: $v(t) = v_0 + \int_0^t a(t)\,dt = \frac{1}{2}Pt^2\,\hat{i} + Qt\,\hat{j}$

Position: $r(t) = r_0 + \int_0^t v(t)\,dt = \frac{1}{6}Pt^3\,\hat{i} + \frac{1}{2}Qt^2\,\hat{j}$

Now, if the particle passes through the point (a, a), then

$$\frac{1}{6}Pt^3 = a \quad \text{and} \quad \frac{1}{2}Qt^2 = a$$

Both the above equations should give us the same value of t

Therefore, $\left(\frac{6a}{P}\right)^2 = \left(\frac{2a}{Q}\right)^3$ $\Rightarrow$ $a = \frac{9Q^3}{2P^2}$

Also, solving for t from both the equations separately,

$$t = \left(\frac{6a}{P}\right)^{1/3} \quad \text{and} \quad t = \left(\frac{2a}{Q}\right)^{1/2}$$

At $t = T$, angle made by the velocity of the particle with the positive X-axis,

$$\theta = \tan^{-1}\left(\frac{Qt}{\frac{1}{2}Pt^2}\right) = \tan^{-1}\left(\frac{2Q}{Pt}\right) = \tan^{-1}\left(\frac{2}{3}\right)$$

7.(AC) Since A moves faster than B, clearly it will cover a greater distance before they meet

So, we can look at the situation as A being three-quarters of the circle, i.e. a distance $\frac{3\pi R}{2}$ behind B initially. Hence, the time instant when they meet is given by

$s_A = s_B + \frac{3\pi R}{2}$; $vt = \left(\frac{v}{3}\right)t + \frac{3\pi R}{2}$

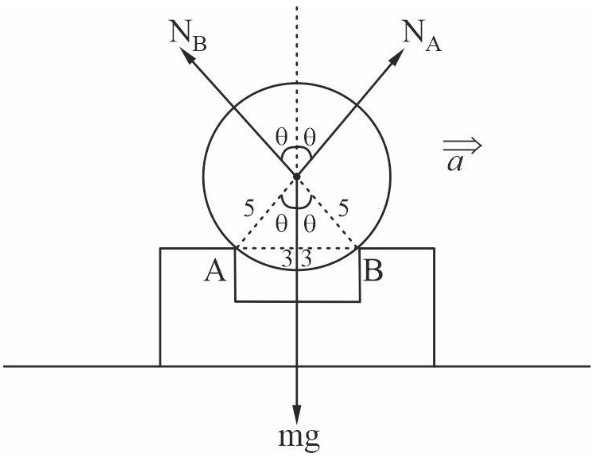
$\Rightarrow$ $t = \frac{9\pi R}{4v}$ $T_1 = \frac{9\pi R}{4v}$

Let the time elapsed after $t = T_1$ until the particles meet again be t

Then, $s_A = s_B + 2\pi R$; $vt = \left(\frac{v}{3}\right)t + 2\pi R$

$\Rightarrow$ $t = \frac{3\pi R}{v}$ So, $T_2 = T_1 + \frac{3\pi R}{v} = \frac{21\pi R}{4v}$

8.(CD)



From the figure: $N_A \cos\theta + N_B \cos\theta = mg$

$N_A \sin\theta - N_B \sin\theta = ma$

Solving, we get $N_A = \frac{1}{2}\left(\frac{mg}{\cos\theta} + \frac{ma}{\sin\theta}\right)$

Also from the figure: $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$

So, $N_A = 95/3 \text{ N}$; $N_B = 55/3 \text{ N}$

9.(ABD) Before the string breaks, the acceleration of both blocks,

$a = \left(\frac{2.1-1.9}{2.1+1.9}\right)g = 0.5 \text{ m/s}^2$ upward for block A and downward for block B

Therefore, at $t = 1.0$ s, the velocities of the blocks are: (taking upward positive)

$v_{A1} = (0.5)(1) = 0.5 \text{ m/s}$; $v_{B1} = (-0.5)(1) = -0.5 \text{ m/s}$

After the string breaks, acceleration of both blocks is g downwards

So, at $t = 1.1$ s, the velocities of the blocks are:

$v_{A2} = 0.5 + (-10)(0.1) = -0.5 \text{ m/s}$; $v_{B2} = -0.5 + (-10)(0.1) = -1.5 \text{ m/s}$

Now, between $t = 0$ and $t = 1.1$ s, the total displacements of the two blocks are:

$$s_A = \left(\frac{1}{2}(0.5)(1)^2\right) + \left((0.5)(0.1) + \frac{1}{2}(-10)(0.1)^2\right) = 0.25 \text{ m} = 25 \text{ cm}$$

$$s_B = \left(\frac{1}{2}(-0.5)(1)^2\right) + \left((-0.5)(0.1) + \frac{1}{2}(-10)(0.1)^2\right) = -0.35 \text{ m} = -35 \text{ cm}$$

10.(BCD) Here $\alpha = (3/r)$, $\omega = \omega_0 + \alpha t = \alpha t$

Also, $\omega^2 = \omega_0^2 + 2\alpha\theta$

so, $\theta = \frac{\omega^2}{2\alpha}$

But $\omega^2 r = 3$ (when $a_t = a_r$)

so, $\omega^2 = \frac{3}{r}$

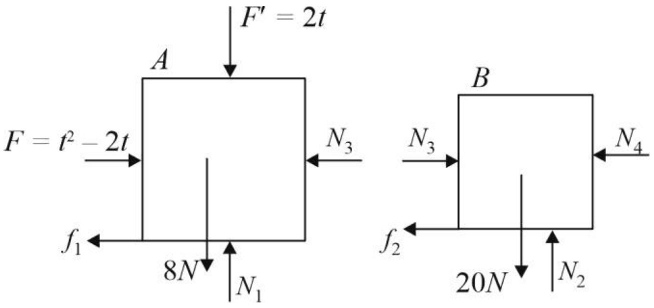
so, $\theta = \frac{3/r}{2(3/r)} = \frac{1}{2}$

$$a_{net} = \sqrt{a_t^2 + a_r^2} = 3\sqrt{2}\ \text{m/s}^2$$

$$t = \frac{\omega}{\alpha} = \frac{\sqrt{3/r}}{3/r} = \sqrt{\frac{r}{3}} = \sqrt{\frac{50}{3}}\ \text{sec}$$

$$s = \frac{1}{2}a_t t^2 = \frac{1}{2}3\left(\frac{50}{3}\right) = 25m$$

11.(C) **12.(D)**



For block A:

$$\sum F_y = 0 \Rightarrow N_1 = 2t + 8N$$

So, $(f_1)_{\max} = \mu_1 N_1 = (0.5)(2t+8)$ $\Rightarrow$ $(f_1)_{\max} = t + 4N$

Block A will press block *B* after $F = (f_1)_{\max}$

$\Rightarrow$ $t^2 - 2t = t + 4$ $\Rightarrow$ $t^2 - 3t - 4 = 0$ $\Rightarrow$ $t = 4s$

So, for $0s \le t \le 4s,\ f_1 = F$

$\Rightarrow$ $f_1 = t^2 - 2t$ for $0s \le t \le 4s$

$\Rightarrow$ f_1 vs t graph will be a parabola

As $t = 2s$, $f_1 = (2)^2 - 2(2)$ $\Rightarrow$ $f_1 = 0N$

And for $t > 4s,\ f_1 = t + 4N$

Hence $N_3 = F - f_1 = (t^2 - 2t) - (t + 4)$

$\Rightarrow$ $N_3 = t^2 - 3t - 4N$

For Block B:

$$\sum F_y = 0 \Rightarrow N_2 = 20N$$

So, $(f_2)_{\max} = \mu_2 N_2 = (0.7)(20)$ $\Rightarrow$ $(f_2)_{\max} = 14N$

Block B will press wall after $N_3 = (f_2)_{\max}$

$\Rightarrow$ $t^2 - 3t - 4 = 14$

$\Rightarrow$ $t^2 - 3t - 18 = 0 \Rightarrow$ $t = 6s$

For $4s \le t \le 6s,$ $f_2 = N_3$

$\Rightarrow$ $f_2 = t^2 - 3t - 4N$

Hence at $t = 5s$

$f_2 = (5)^2 - 3(5) - 4 = 6N$

SECTION 2

1.(0.25)

Let the X-components of initial velocities be $\vec{u}_{x1}$ and $\vec{u}_{x2}$

Let the Y-components of initial velocities be $\vec{u}_{y1}$ and $\vec{u}_{y2}$

Now, we know that the relative X-displacement in the first 1.2 seconds is 24 m, and the relative Y-displacement in the first 1.2 seconds is 6 m

So, $\left|\left(\vec{u}_{x1}-\vec{u}_{x2}\right)\right|(1.2)=24$

And, $\left|\left(\vec{u}_{y1}(1.2)-\frac{1}{2}g(1.2)^2\right)-\left(\vec{u}_{y2}(1.2)-\frac{1}{2}g(1.2)^2\right)\right|=6 \Rightarrow \left|\left(\vec{u}_{y1}-\vec{u}_{y2}\right)\right|(1.2)=6$

(It does not matter which particle we call 1 and which we call 2, as that will only change the sign of the X and Y components of the relative velocity, and not change the angle the relative velocity makes with the horizontal)

We get $\left|\vec{u}_{x1}-\vec{u}_{x2}\right|=20$ m/s and $\left|\vec{u}_{y1}-\vec{u}_{y2}\right|=5$ m/s

Now, since the acceleration of the particles is the same (*g* downwards), their relative velocity remains constant while they are both in the air

Hence, $\tan\theta=\frac{\left|\vec{u}_{y1}-\vec{u}_{y2}\right|}{\left|\vec{u}_{x1}-\vec{u}_{x2}\right|}=\frac{1}{4}$

2.(9) Let the acceleration of the blocks be *a*

Then, the minimum value of *a* such that block B slips on A is

$$a_{\min}=(0.2)g \Rightarrow a_{\min}=2 \text{ m/s}^2$$

Now, for the system of the two blocks together,

$$F-(0.1)(3g)=3a \Rightarrow F=3a+3$$

Therefore, for slipping between A and B,

$$F_{\min}=3a_{\min}+3=3(2)+3=9 \text{ N}$$

3.(4) $0=v_0\cos 30-g\sin 30t \Rightarrow t=\frac{v_0\cos 30}{g\sin 30}$ ... (i)

$-H\cos 30=-v_0\sin 30t-\frac{1}{2}g\cos 30t^2$ ... (ii)

From (i) and (ii)

$$H=\frac{v_0^2}{g}\left[1+\frac{\cot^2 30°}{2}\right] \Rightarrow v_0=\sqrt{\frac{2gH}{5}}=4$$

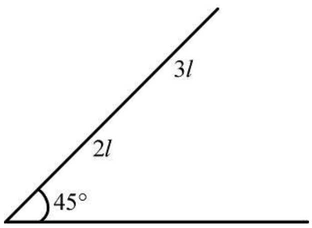
4.(1) $\vec{v}_1=-v_1\hat{i}-gt\,j$; $\vec{v}_2=+v_2\hat{i}-g\,t\,j$

$\vec{v}_1.\vec{v}_2=0$ $-v_1.v_2+g^2t^2=0$

$$t=\frac{\sqrt{v_1v_2}}{g}$$

The particles will be parallel to *x*-axis. Separation will be $x_1+x_2=\frac{(v_1+v_2)\sqrt{v_1v_2}}{g}=1m$

5.(5) While the block reaches down to bottom. Its potential energy is lost due to friction



For smooth part

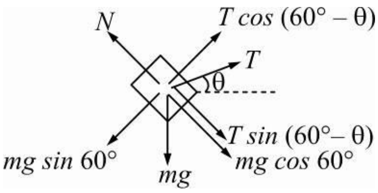
$$v^2 - 0^2 = 2(g\sin\theta)(3l) \quad \ldots \text{(i)}$$

For rough part

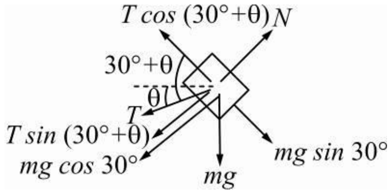
$$0^2 - v^2 = 2(g\sin\theta - \mu g\cos\theta)(2l) \quad \ldots \text{(ii)}$$

Dividing (i) and (ii) we get $\mu = \frac{5}{2}$

6.(30) Forces on block A and B



Block A



Block B

So for equilibrium of A and B along inclination of wedge

$$T\cos(60° - \theta) = mg\sin 60° \quad \ldots \text{(i)}$$

$$T\cos(30 + \theta) = mg\sin 30° \quad \ldots \text{(ii)}$$

Divide (i) by (ii)

$$\frac{\cos(60° - \theta)}{\cos(30° + \theta)} = \frac{\sqrt{3}}{1}$$

So $\theta = 30°$

[CHEMISTRY]

1.(D) **(A)** $2IF_5 \longrightarrow IF_4^+ + IF_6^-$
(sp^3d^2) (sp^3d) (sp^3d^3)

(B) $I_2Cl_6 \longrightarrow ICl_2^+ + ICl_4^-$
(sp^3d^2) (sp^3) (sp^3d^2)

(C) $CH_3 - CH_3 \longrightarrow CH_3^+ + CH_3^-$
(sp^3) (sp^3) (sp^2) (sp^3)

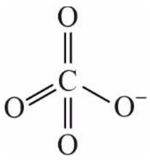
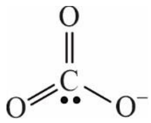
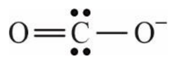
(D) $(CH_3)_3N + H^+ \longrightarrow (CH_3)_3NH^+$
(sp^3) (sp^3)

2.(C) Out of the given elements

(I) Most E.N. $\rightarrow$ F **(II)** Maximum hydration energy $\rightarrow Li^+$

(III) Max. I.E. $\rightarrow$ Ne **(IV)** Most electropositive $\rightarrow$ Cs

3.(C)

	ClO_4^-	ClO_3^-	ClO_2^-
			
Cl-atom :	sp^3	sp^3	sp^3
π bonds :	Three	Two	One
	$Cl_{(d)} \overset{\pi}{—} O_{(p)}$	$Cl_{(d)} \overset{\pi}{—} O_{(p)}$	$Cl_{(d)} \overset{\pi}{—} O_{(p)}$
Avg. Cl – O bond order :	7/4	5/3	3/2
Magnetic nature :	Diamagnetic	Diamagnetic	Diamagnetic

4.(B) Unt : un nil trium, Z = 103, Belongs to 7th period, 3rd group
Uub : un un bium, Z = 112, Belongs to 7th period, 12th group
Z = 112 has zero unpaired electron in penultimate d-subshell.
Z = 103 has one unpaired electron in penultimate d-subshell.

5.(C) Valency of anion of a non-metal of 15th group = –3
Valency of anion of a non-metal of 16th group = –2
Valency of anion of a non-metal of 17th group = –1
Formation of –1 anion is exothermic while formation of –2 and –3 anion of elements is highly endothermic.
Lattice energy of salts with anion of 15th and 16th group will be greater than lattice energy of salt with anion of 17th group. Lattice enthalpy will be negative with large magnitude for all salts.

6.(ACD)
Cl – Cl bond has higher bond energy than F – F bond due to repulsion between lone pair of two F-atom.
$C \equiv O$ is stronger than O = O.
Due to bigger atomic size of Br atoms Br – Br bond is longer than F – F bond.

7.(AD) $Xe \xrightarrow{O_2F_2} XeF_2 + XeF_4 + XeF_6$

	XeF_2	XeF_4	XeF_6
(Geometry) :	Linear	Sq. planar	Distorted octahedral
(Hybridisation) :	sp^3d	sp^3d^2	sp^3d^3
(Lone pair over Xe) :	3	2	1
(Compound) :	P	R	Q

8.(AD) Correct orders of electron affinities

(A) O < F **(B)** F < Cl **(C)** Li > Be **(D)** N < O

9.(ACD)

A, C and D statements are correct regarding the long form of the periodic table.

10.(BC)

The given trend of ionisation enthalpy is for

$I \to Be$; $II \to O$; $III \to Al$; $IV \to Ga$; $V \to Se$

11.(A) For stationary dipole-dipole interactions; $V_{(r)} \propto \frac{1}{r^3}$

12.(B) Polarizability of a particle increases with size thus correct orders are:

(A) $CH_4 < SiH_4 < GeH_4$ **(B)** $F_2 < Cl_2 < Br_2$

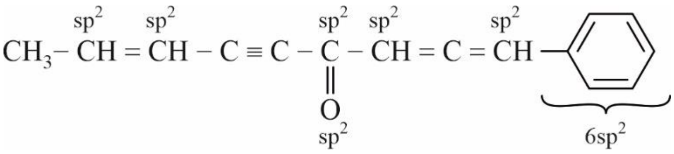
(D) He < Ne < Ar

Among dipoles H – X, boiling point increases with molar mass.

$\therefore$ Boiling point order : H – Cl < H – Br < H – I

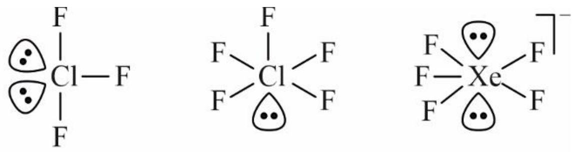
SECTION 2

1.(12)



$\therefore$ Number of sp^2 atom = 12

2.(18)



Number of 90° angles between L.P and B.P. 4 4 10

3.(269) $\chi_{Cl} - \chi_H = 0.1(\Delta)^{1/2}$

$3 - 2.1 = 0.1(\Delta)^{1/2}$

$0.9 = 0.1(\Delta)^{1/2} \quad \Rightarrow \quad \Delta = 81$

$81 = E_{H-Cl} - \frac{1}{2}\left[E_{H-H} + E_{Cl-Cl}\right]$

$81 = E_{H-Cl} - \frac{1}{2}[400 + 300] \quad \Rightarrow \quad E_{H-Cl} = 269\,kJ/mole$

4.(7) 7 polar molecules : PCl_2F_3, SO_2, CH_3Cl, $CHCl_3$, OF_2, NCl_3 and C_6H_5Cl.

5.(269) $E.A = (P.E)_H - (P.E)_{H^-} = 2.8\,eV = 2.8 \times 96.2 = 269.36\,kJ/mole$

6.(7) All of the given statements are true.

[MATHEMATICS]

SECTION 1

1.(B) $\cos^2\frac{\pi}{8} = \frac{\sqrt{2}+1}{2\sqrt{2}} = \frac{1}{2}+\frac{1}{2\sqrt{2}}$

Now other root is conjugate of this: $\Rightarrow \quad \frac{\sqrt{2}-1}{2\sqrt{2}} = \frac{1}{2}-\frac{1}{2\sqrt{2}}$

$\therefore$ Sum of roots = $-b = 1 \quad \Rightarrow \quad b = -1$

Product of root = $c = \frac{1}{8} \quad \Rightarrow \quad (b, c) = \left(-1, \frac{1}{8}\right)$

2.(A) $\cos^2\theta = \frac{x^2+y^2+1}{2x} \quad \Rightarrow \quad 0 \le \cos^2\theta \le 1 \quad \Rightarrow \quad 0 \le \frac{x^2+y^2+1}{2x} \le 1$

If $\frac{x^2+y^2+1}{2x} \ge 0 \quad \Rightarrow \quad x > 0$

If $\frac{x^2+y^2+1}{2x} - 1 \le 0 \quad \because \quad x > 0 \quad \Rightarrow \quad (x-1)^2 + y^2 \le 0 \quad \Rightarrow \quad x = 1,\ y = 0$

3.(B) Let common ratio of GP be r.

$b_1 = 1$, $b_2 = r$ and $b_3 = r^2$

$\therefore \quad 4b_2 + 5b_3 = 5r^2 + 4r \quad = 5\left[\left(r+\frac{2}{5}\right)^2 - \frac{4}{25}\right] = 5\left(r+\frac{2}{5}\right)^2 - \frac{4}{5}$

Minimum value is $-\frac{4}{5}$, occurs at $r = \frac{-2}{5}$

4.(C) If d is the common difference then $a_1 - a_2 = a_2 - a_3 = \ldots = a_{n-1} - a_n = -d$

Given expression $= \left(\sqrt{a_1}+\sqrt{a_n}\right)\left[\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right]$

$$= \left(\sqrt{a_1}+\sqrt{a_n}\right)\left[\frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} + \ldots + \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n}\right]$$

$$= \frac{\sqrt{a_1}+\sqrt{a_n}}{-d}\left[\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}-\sqrt{a_3}+\ldots+\sqrt{a_{n-1}}-\sqrt{a_n}\right] = -\left(\sqrt{a_1}+\sqrt{a_n}\right)\left(\sqrt{a_1}-\sqrt{a_n}\right)/d$$

$$= -\frac{(a_1-a_n)}{d} = \frac{(a_n-a_1)}{d} = \frac{(n-1)d}{d} = n-1$$

5.(C) $\sum a_i = 10 \times \frac{(2+3)}{2} = 25; \qquad \sum \frac{1}{h_i} = 10 \times \frac{\left(\frac{1}{2}+\frac{1}{3}\right)}{2} = \frac{25}{6}$

$(g_1 g_2 \ldots g_{10}) = (2\times 3)^5 \Rightarrow (g_1 g_2 \ldots g_{10})^{1/5} = 6$

$\Rightarrow$ The required product is $= 25 \times \frac{25}{6} \times 6 = 625$

6.(AC) $\dfrac{\dfrac{n(n+1)}{2}-(2K+1)}{n-2}=\dfrac{105}{4}$ (Let $x_1=K,\ x_2=K+1$)

$2n(n+1)-(8K+4)=105n-210$; $\quad 2n^2-103n-8K+206=0$

$2n^2-103n+206=8K\in[8,\,8(n-1)]$ as $K\in[1,\,n-1]$

$\Rightarrow \quad 8\le 2n^2-103n+206\le 8(n-1)$

$\Rightarrow \quad 2n^2-103n+206\ge 8$ and $2n^2-103n+206\le 8n-8$

$\Rightarrow \quad 2n^2-103n+198\ge 0$ and $2n^2-111n+214\le 0$

$\Rightarrow \quad 2n^2-4n-99n+198\ge 0$ and $2n^2-4n-107n+214\le 0$

$\Rightarrow \quad (2n-99)(n-2)\ge 0$ and $(2n-107)(n-2)\le 0$

$\Rightarrow \quad n\le 2$ or $n\ge 49.5$ and $2\le n\le 53.5$

$\Rightarrow \quad n=50$ {$n-2$ must be a multiple of four because average of remaining numbers is $105/4$}

For $\quad n=50,\ K=7$

$\Rightarrow \quad x_1=7,\ x_2=8,\ n=50$

Product of remaining members $=\dfrac{50!}{7\times 8}$

7.(ABCD) $\dfrac{1}{a},\dfrac{1}{b},\dfrac{1}{c}$ are in A.P.

$\Rightarrow \quad \dfrac{a+b+c}{a},\dfrac{a+b+c}{b},\dfrac{a+b+c}{c}$ are in A.P.

$\Rightarrow \quad \dfrac{a+b+c}{a}-2,\ \dfrac{a+b+c}{b}-2,\ \dfrac{a+b+c}{c}$ are in A.P.

Hence option (A) is correct. Similarly option (B) is correct

Since a, b, c > 0 are distinct $\quad\Rightarrow\quad \dfrac{a^5+c^5}{2}>(a^5c^5)^{1/2}$

$\Rightarrow \quad \dfrac{a^5+c^5}{2}>((ac)^{1/2})$ also $(ac)^{1/2}>b$ (GM > HM)

$\Rightarrow \quad a^5+c^5>2b^5$ which is included in (C)

Hence option C is correct

Also $\quad \dfrac{a-b}{b-c}=\dfrac{a}{c}\Rightarrow ac-bc=ab-ac$

$b=\dfrac{2ac}{a+c} \quad\Rightarrow\quad$ Option D is correct

8.(AC) $\sin\theta+\sin 7\theta+\sin 4\theta=0 \quad\Rightarrow\quad 2\sin 4\theta\cos 3\theta+\sin 4\theta=0$

$\Rightarrow \quad \sin 4\theta=0$ or $\cos 3\theta=\dfrac{-1}{2}=\cos\left(\dfrac{2\pi}{3}\right)$

$\Rightarrow \quad 4\theta=n\pi \Rightarrow \theta=\dfrac{n\pi}{4}=\dfrac{\pi}{4},\dfrac{\pi}{2},\dfrac{3\pi}{4}$

or $\quad 3\theta=2n\pi\pm\dfrac{2\pi}{3}$ i.e. $\theta=\dfrac{2n\pi}{3}\pm\dfrac{2\pi}{9}$ i.e. $\dfrac{2\pi}{9},\dfrac{8\pi}{9},\dfrac{4\pi}{9}$

9.(AB) We have $A_1 = \dfrac{3a+b}{4},\ A_2 = \dfrac{a+b}{2},\ A_3 = \dfrac{a+3b}{4}$

$$G_1 = \left(a^3b\right)^{1/4},\ G_2 = (ab)^{1/2},\ G_3 = \left(ab^3\right)^{1/4};\quad H_1 = \frac{4ab}{(a+3b)},\ H_2 = \frac{2ab}{(a+b)},\ H_3 = \frac{4ab}{(3a+b)}$$

$$\Rightarrow \quad A_2H_2 = ab = G_2^2$$

$$G_2^2 = A_1H_3 = A_2H_2 = A_3H_1 = ab$$

10.(AD) $x = \dfrac{a+b}{2},\ y = \dfrac{b+c}{2},\ b^2 = ac$ is given

$$\Rightarrow \quad \frac{1}{x}+\frac{1}{y} = 2\left(\frac{1}{a+b}+\frac{1}{b+c}\right) = \frac{2(a+c+2b)}{(a+b)(b+c)} = \frac{2(a+2b+c)}{b^2+ac+ab+bc} = \frac{2(a+2b+c)}{b(a+2b+c)} = \frac{2}{b}$$

$$\frac{a}{x}+\frac{c}{y} = \frac{2a}{a+b}+\frac{2c}{b+c} = \frac{2(ab+ac+ca+cb)}{(a+b)(b+c)} = \frac{2b(a+2b+c)}{b(a+2b+c)} = 2 \qquad \{\text{Using } b^2 = ac\}$$

11.(C) **12.(B)**

$$T_r = \frac{8r}{4r^4+1} = \frac{8r}{\left(4r^4+4r^2+1\right)-4r^2} = \frac{8r}{\left(2r^2+1\right)^2-(2r)^2}$$

$$= \frac{8r}{\left(2r^2-2r+1\right)\left(2r^2+2r+1\right)} = 2\left[\frac{1}{\left(2r^2-2r+1\right)} - \frac{1}{\left(2r^2+2r+1\right)}\right]$$

$$S_n = 2\left[1-\frac{1}{2n^2+2n+1}\right] \qquad \Rightarrow \qquad S_\infty = 2 \text{ and } \qquad S_{16} = \frac{1088}{545}$$

SECTION 2

1.(3) $S_n = cn(n+1) = cn^2+cn$

$$S_{n-1} = c(n-1)^2 + c(n-1)$$

$$t_n = S_n - S_{n-1} = c(2n-1)+c = c.2n$$

$$t_n^2 = c^2 4n^2$$

$$\sum t_n^2 = c^2 4.\frac{(n)(n+1)(2n+1)}{6} = \frac{2}{3}c^2(n)(n+1)(2n+1)$$

2.(8) $\dfrac{\tan 20^\circ + \tan 40^\circ + \tan 80^\circ - \tan 60^\circ}{\sin 40^\circ}$

$$= \left(\frac{\sin 60^\circ}{\cos 20^\circ \cos 40^\circ} + \frac{\sin 20^\circ}{\cos 80^\circ \cos 60^\circ}\right)\frac{1}{\sin 40^\circ}$$

$$= \frac{\sin 60^\circ \cos 60^\circ \cos 80^\circ + \sin 20^\circ \cos 20^\circ \cos 40^\circ}{\cos 20^\circ \cos 40^\circ \cos 80^\circ \cos 60^\circ \sin 40^\circ}$$

$$= \frac{2\sin 120^\circ \cos 80^\circ + \sin 80^\circ}{4\times 1/8 \times 1/2 \times \sin 40^\circ} = 8\times\frac{\frac{\sqrt{3}}{2}\cos 80^\circ + \frac{1}{2}\sin 80^\circ}{\sin 40^\circ} = 8\times\frac{\sin 140^\circ}{\sin 40^\circ} = 8$$

3.(4) Number of points of intersection is given by solutions of $f(x)=g(x)$

$\Rightarrow \quad \sin 3x + \cos x = \cos 3x + \sin x \quad \Rightarrow \quad \sin 3x - \sin x = \cos 3x - \cos x$

$2\cos 2x . \sin x = -2\sin 2x . \sin x \quad \Rightarrow \quad \sin x = 0$ or $\tan 2x = -1$

So on interval $[0, \pi]$

$$x = 0, \pi, \frac{3\pi}{8}, \frac{7\pi}{8}$$

4.(0) $|\sin x \cos x| + \sqrt{\tan^2 + \cot^2 x + 2} = \sqrt{3}$

$\Rightarrow \quad |\sin x \cos x| + |\tan x + \cot x| = \sqrt{3} \quad \Rightarrow \quad |\sin x \cos x| + \left|\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right| = \sqrt{3}$

$\Rightarrow \quad |\sin x \cos x| + \frac{1}{|\sin x \cos x|} = \sqrt{3}$

Let $|\sin x \cos x| = t,$ then $t + \frac{1}{t} = \sqrt{3}$ where $t > 0$

But $t + \frac{1}{t} = \left(\sqrt{t} - \frac{1}{\sqrt{t}}\right)^2 + 2 \geq 2$

Hence, $t + \frac{1}{t}$ can not be equal to $\sqrt{3}.$

5.(9) $S = \sin\frac{\pi}{7}\sin\frac{3\pi}{7} + \sin\frac{3\pi}{7}\sin\frac{5\pi}{7} + \sin\frac{5\pi}{7}\sin\frac{\pi}{7}$

$S = \sin\frac{\pi}{7}\left[\sin\frac{3\pi}{7} + \sin\frac{5\pi}{7}\right] + \sin\frac{3\pi}{7}\sin\frac{5\pi}{7}$

$S = \sin\frac{\pi}{7}\left[2\sin\frac{4\pi}{7}\cos\frac{\pi}{7}\right] + \sin\frac{3\pi}{7}\sin\frac{2\pi}{7}$

$S = \sin\frac{2\pi}{7}\sin\frac{4\pi}{7} + \sin\frac{2\pi}{7}\sin\frac{3\pi}{7}$

$S = 2\sin\frac{2\pi}{7}\sin\frac{4\pi}{7} = \cos\frac{2\pi}{7} - \cos\frac{6\pi}{7} = \left(2\cos^2\frac{\pi}{7} - 1\right) - \left(-\cos\frac{\pi}{7}\right) = 2\cos^2\frac{\pi}{7} + \cos\frac{\pi}{7} - 1$

$$f\left(\cos\frac{\pi}{7}\right) = 2\cos^2\frac{\pi}{7} + \cos\frac{\pi}{7} - 1$$

$\Rightarrow \quad f(x) = 2x^2 + x - 1 \quad \Rightarrow \quad f(2) = 9$

6.(2) $\sum_{r=1}^{5}\frac{1}{r(r+1)(r+2)(r+3)} = \frac{1}{3}\sum_{r=1}^{5}\left(\frac{1}{r(r+1)(r+2)} - \frac{1}{(r+1)(r+2)(r+3)}\right)$

$$= \frac{1}{3}\left[\frac{1}{1\cdot 2\cdot 3} - \frac{1}{6\cdot 7\cdot 8}\right] = \frac{1}{18} - \frac{1}{18\cdot 56} = x$$

$54x = 3 - \frac{3}{56}$